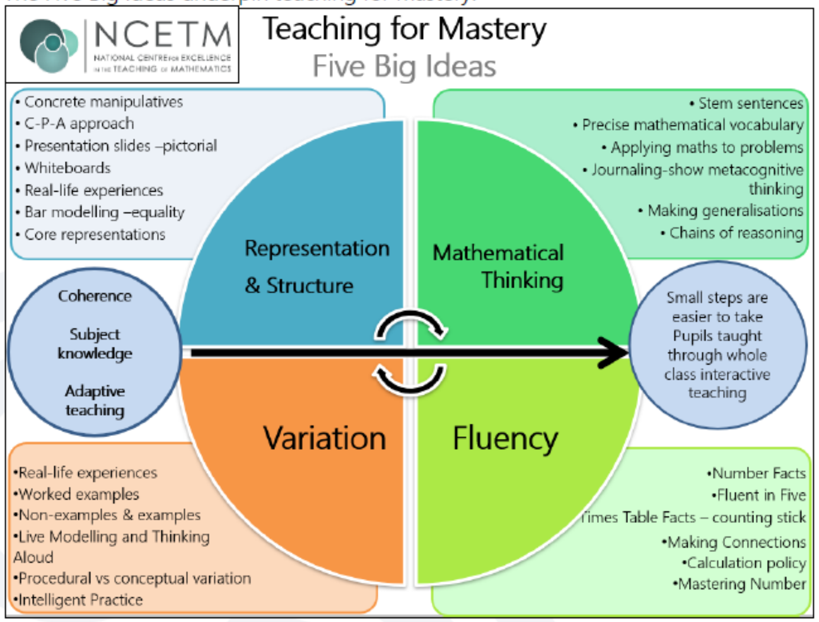
Mathematics Calculation Policy 2025-2026

At Witham Oaks Academy, we use the resources from The NCETM as the basis for our planning. We follow the 'concrete – pictorial – abstract' approach to modelling calculations to ensure children have a sustainable and deep understanding of the maths before using the abstract calculation method. This calculation policy is based on the Dfe Guidance *'Mathematics guidance: key stages 1 and 2'* and is split into two sections, addition and subtraction, and multiplication and division.

This policy (is):

- based on research evidence about the most effective ways to teach mathematics to enhance children's understanding (eg Bruner's modes of representation that have morphed into Concrete Pictorial Abstract)
- based on substantive research evidence (Haylock, Anghileri, Thompson, Askew, Boaler, etc) that procedural fluency and conceptual understanding must be taught together
- based on the 'Five Big Ideas' in teaching for mastery – that mathematical knowledge and understanding is incremental and thus it is essential to "master" each step otherwise gaps in learning will compromise future success. Children will not be accelerated onto content from future years but will be extended through problem solving.

The Five Big Ideas underpin teaching for mastery.



Place Value

Year 2

2NPV–1 Place value in two-digit numbers

Pupils need to be able to connect the way two-digit numbers are written in numerals to their value. They should demonstrate their reasoning using full sentences.

Pupils should recognise that 42, for example, can be composed either of 42 ones, or of 4 tens and 2 ones. They should be able to group objects into tens, with some left over ones, to count efficiently and to demonstrate an understanding of the number. Pupils need to be capable of identifying the total quantity in different representations of groups of ten and additional ones. Within these representations the relative positions of the tens and the ones should be varied.

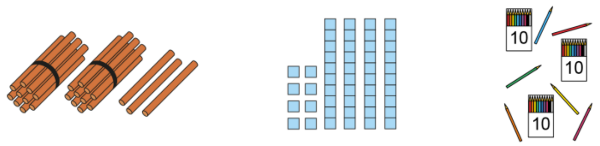


Figure 41: varied representations of two-digit numbers as groups of ten and additional ones

Pupils need to be able to partition two-digit numbers into tens and ones parts, and represent this using diagrams, and addition and subtraction equations.

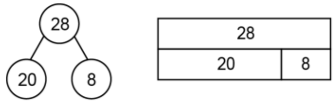


Figure 42: partitioning 28 into 20 and 8

$20 + 8 = 28$ $\quad 28 - 20 = 8$

$8 + 20 = 28$ $\quad 28 - 8 = 20$

$28 = 20 + 8$ $\quad 8 = 28 - 20$

$28 = 8 + 20$ $\quad 20 = 28 - 8$

It is also important for pupils to be able to think flexibly about number, learning to:

- partition into a multiple of ten and another two-digit number, in different ways (for example, 68 can be partitioned into 50 and 18, into 40 and 28, and so on)
- partition into a two-digit number and a one-digit number, in different ways (for example, 68 can be partitioned into 67 and 1, 66 and 2, and so on)

Year 3

3NPV–2 Place value in three-digit numbers

Recognise the place value of each digit in three-digit numbers, and compose and decompose three-digit numbers using standard and non-standard partitioning.

Pupils should be able to identify the place value of each digit in a three-digit number. They must be able to combine units of ones, tens and hundreds to compose three-digit numbers, and partition three-digit numbers into these units. Pupils need to experience variation in the order of presentation of the units, so that they understand that 40 + 300 + 2 is equal to 342, not 432.

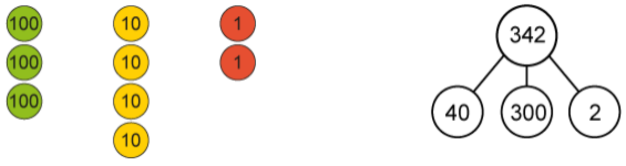


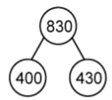
Figure 65: two representations of the place-value composition of 342

Pupils also need to solve problems relating to subtraction of any single place-value part from the whole number, for example:

$$342 - 300 = \square$$

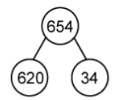
$$342 - \square = 302$$

As well as being able to partition numbers in the 'standard' way (into individual placevalue units), pupils must also be able to partition numbers in 'non-standard' ways, and carry out related addition and subtraction calculations, for example:



830 – 400 = 430

Figure 66: partitioning 830 into 430 and 400



620 + 34 = 654

Figure 67: partitioning 654 into 620 and 34

Year 4

4NPV–2 Place value in four-digit numbers

Recognise the place value of each digit in four-digit numbers, and compose and decompose four-digit numbers using standard and non-standard partitioning.

Pupils should be able to identify the place value of each digit in a four-digit number. They must be able to combine units of ones, tens, hundreds and thousands to compose fourdigit numbers, and partition four-digit numbers into these units. Pupils need to experience variation in the order of presentation of the units, so that they understand that 40 + 300 + 2 + 5000 is equal to 5,342, not 4,325.

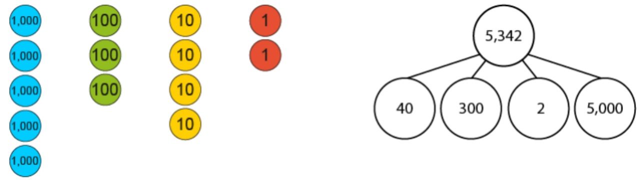


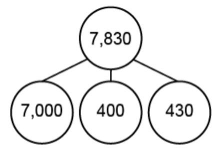
Figure 106: 2 representations of the place-value composition of 5,342

Pupils also need to solve problems relating to subtraction of any single place-value part from the whole number, for example:

$$5{,}342 - 300 = \square$$

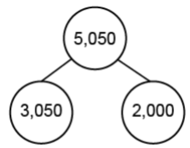
$$5{,}342 - \square = 5{,}302$$

As well as being able to partition numbers in the 'standard' way (into individual place value units), pupils must also be able to partition numbers in 'non-standard' ways, and carry out related addition and subtraction calculations, for example:



7,830 – 400 = 7,430

Figure 107: partitioning 7,830 into 7,430 and 400



2,000 + 3,050 = 5,050

Figure 108: partitioning 5,050 into 2,000 and 3,050

Year 5

5NPV–2 Place value in decimal fractions

Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and nonstandard partitioning.

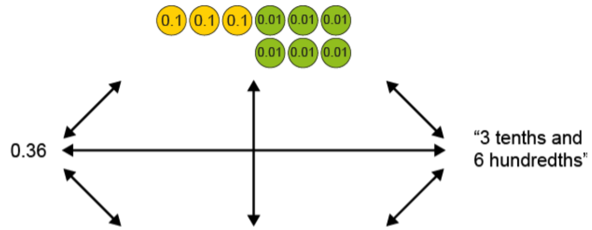
Pupils must be able to read, write and interpret decimal fractions with up to 2 decimal places. Pupils should work first with decimal fractions with one significant digit (for example, 0.3 and 0.03). The Gattegno chart is a useful tool here.

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Figure 157: Gattegno chart showing thousands, hundreds, tens, ones, tenths and hundredths

The number 300 is spoken as “three hundred” rather than as “three-zero-zero”, and this helps pupils to identify the value of the 3 in 300. However, decimal fractions are usually spoken as digits, for example, 0.03 is spoken as “zero-point-zero-three” (or “noughtpoint-nought-three”) rather as “three hundredths”. As such, pupils need to practise speaking decimal fractions in both ways and learn to convert from one to the other.

Pupils must then learn to work with decimal fractions with 2 significant digits (for example, 0.36). For any given decimal fraction of this type, pupils must be able to connect the spoken words (zero-point-three-six), the value in decimal notation (0.36), describing the number of tenths and hundredths (3 tenths and 6 hundredths) and visual representations (such as place-value counters and the Gattegno chart).



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Figure 158: 4 different represenations of 0.36

Pupils should be able to identify the place value of each digit in numbers with up to 2 decimal places. They must be able to combine units of hundredths, tenths, ones, tens, hundreds and thousands to compose numbers, and partition numbers into these units. Pupils need to experience variation in the order of presentation of the units, so that they understand that 0.4 + 3 + 0.02 + 50 is equal to 53.42, not 43.25.

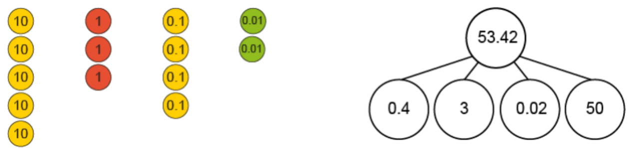
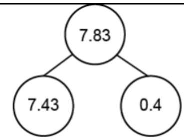


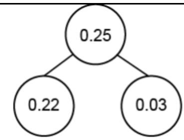
Figure 159: 2 representations of the place-value composition of 53.42

As well as being able to partition numbers in the ‘standard’ way (into individual placevalue units), pupils must also be able to partition numbers in ‘non-standard’ ways and carry out related addition and subtraction calculations, for example:



7.83 – 0.4 = 7.43

Figure 160: partitioning 7.83 into 7.43 and 0.4



0.22 + 0.03 = 0.25

Figure 161: partitioning 0.25 into 0.22 and 0.03

Year 6		
6NPV–2 Place value in numbers up to 10,000,000 Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning. Pupils must be able to read and write numbers up to 10,000,000, including decimal fractions. Pupils should be able to use a separator (such as a comma) every third digit from the decimal separator to help read and write numbers. Pupils must be able to copy numbers from calculator displays, inserting thousands separators and decimal points correctly. This will prepare them for secondary school, where pupils will be expected to know how to use calculators. Pupils need to be able to identify the place value of each digit in a number. Pupils must be able to combine units from millions to hundredths to compose numbers, and partition numbers into these units, and solve related addition and subtraction calculations. Pupils need to experience variation in the order of presentation of the units, so that they understand, for example, that 5,034,000.2 is equal to 4,000 + 30,000 + 0.2 + 5,000,000. Pupils should be able to represent a given number in different ways, including using place-value counters and Gattegno charts, and write numbers shown using these representations. Pupils should then have sufficient understanding of the composition of large numbers to compare and order them by size. Pupils also need to be able to solve problems relating to subtraction of any single place value part from a number, for example: $381{,}920 - 900 = \boxed{}$ $381{,}920 - \boxed{} = 380{,}920$	As well as being able to partition numbers in the 'standard' way (into individual placevalue units), pupils must also be able to partition numbers in 'non-standard' ways, and carry out related addition and subtraction calculations, for example: $518.32 + 30 = 548.32$ $381{,}920 - 60{,}000 = 321{,}920$ Pupils can initially use place-value counters for support with this type of partitioning and calculation, but by the end of year 6 must be able to partition and calculate without them.	

Year 1

1AS–1 Compose and partition numbers to 10

Pupils should be presented with varied cardinal (quantity) representations, both concrete and pictorial, which support identification of the 'numbers within a number'. The examples below provide different ways of showing that 8 can be composed from 2 numbers.

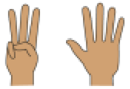


Figure 17: 8 represented as 3 fingers and 5 fingers

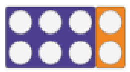


Figure 18: 8 represented as 6 and 2 with base 10 number boards

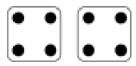


Figure 19: 8 represented as two 4-value dice

Figure 20: 8 represented as 2 rows of 4

Figure 21: 8 represented as tally marks: 5 and 3

Figure 22: 8 represented on a bead string: 7 and 1

Pupils should learn to interpret and sketch partitioning diagrams to represent the ways numbers can be partitioned or combined. At this stage, these should be used alongside quantity images to support development of the understanding of quantity.

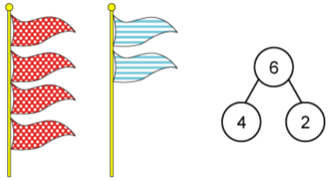


Figure 23: using a partitioning diagram to represent 6 flags, consisting of 4 spotty flags and 2 stripy flags

Pupils should also experience working with manipulatives and practise partitioning a whole number of items into parts, then putting the parts back together.

Year 1

1AS–2 Read, write and interpret additive equations

For each of the 4 additive structures described below (aggregation, partitioning, augmentation and reduction), pupils should learn to link expressions (for example, $5 + 2$ and $6 – 2$) to contexts before they learn to link equations (for example, $5 + 2 = 7$ and $6 – 2 = 4$) to contexts.

1. Pupils should first learn to describe the context using precise language (see the language focus boxes below).
2. Pupils should then learn to write the associated expression or equation.
3. Pupils should then use precise language to describe what each number in the expression or equation represents.

Pupils need to be able to write and interpret expressions and equations to represent aggregation (combining 2 parts to make 1 whole) and partitioning (separating 1 whole into 2 parts).

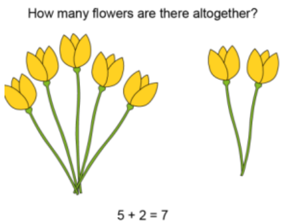


Figure 26: addition as aggregation

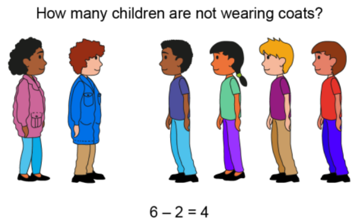


Figure 27: subtraction as partitioning

Pupils must also learn to relate addition and subtraction contexts and equations to mathematical diagrams such as bar models, number lines, tens frames with counters, and partitioning diagrams.

Year 1

1NF–1 I have developed fluency in addition and subtraction facts within 10.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8		
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7			
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6				
5	5+0	5+1	5+2	5+3	5+4	5+5					
6	6+0	6+1	6+2	6+3	6+4						
7	7+0	7+1	7+2	7+3							
8	8+0	8+1	8+2								
9	9+0	9+1									
10	10+0										

The number of addition facts to be learnt is reduced when commutativity is applied and pupils recognise that $3 + 2$, for example, is the same as $2 + 3$.

Examples of appropriate strategies include the following.

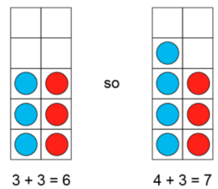


Figure 10: tens frames with counters showing derivation of a 'near-double' addition calculation

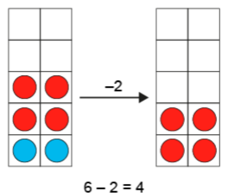


Figure 11: tens frames with counters showing that subtracting 2 from an even number gives the previous even number

Pupils should learn to compose and partition numbers within 10 (1AS–1) before moving on to formal addition and subtraction, using tens frames.

Year 2	Year 2	Year 2

Year 2

2NF–1 I have secured fluency in addition and subtraction facts within 10, through continued practice.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8		
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7			
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6				
5	5+0	5+1	5+2	5+3	5+4	5+5					
6	6+0	6+1	6+2	6+3	6+4						
7	7+0	7+1	7+2	7+3							
8	8+0	8+1	8+2								
9	9+0	9+1									
10	10+0										

The 66 addition facts within 10 are shown on the grid to above. The number of addition facts to be learnt is reduced when commutativity is applied and pupils recognise that 3 + 2, for example, is the same as 2 + 3. Pupils must also have automatic recall of the corresponding subtraction facts, for example 5 – 3 and 5 – 2

- **Composition and securing additive facts: within 5 (2NF-1)**
- **Structures within 10 (2NF-1)**
- **Composition and securing additive facts: 6 to 10 (2NF-1)**
- **Securing additive facts within 10 – strategies (2NF-1)**

Year 2

2NPV–1 Place value in two-digit numbers

Pupils should recognise that 42, for example, can be composed either of 42 ones, or of 4 tens and 2 ones. They should be able to group objects into tens, with some left over ones, to count efficiently and to demonstrate an understanding of the number.

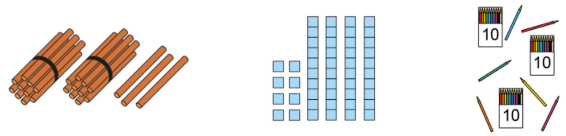


Figure 41: varied representations of two-digit numbers as groups of ten and additional ones

Pupils need to be able to partition two-digit numbers into tens and ones parts, and represent this using diagrams, and addition and subtraction equations.

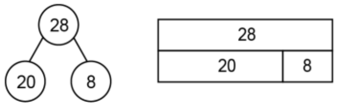


Figure 42: partitioning 28 into 20 and 8

$20 + 8 = 28$ $28 - 20 = 8$
$8 + 20 = 28$ $28 - 8 = 20$

$28 = 20 + 8$ $8 = 28 - 20$
$28 = 8 + 20$ $20 = 28 - 8$

Year 2

<u>2AS–1</u> I can add and subtract across 10

For both addition and subtraction across 10, tens frames and partitioning diagrams can be used to support pupils as they learn about these strategies.

First, pupils should learn to add three one-digit numbers by making 10, for example, 7 + 3 + 2 = 10 + 2. They can then relate this to addition of two numbers across 10, by partitioning one of the addends, for example 7 + 5 = 7 + 3 + 2.

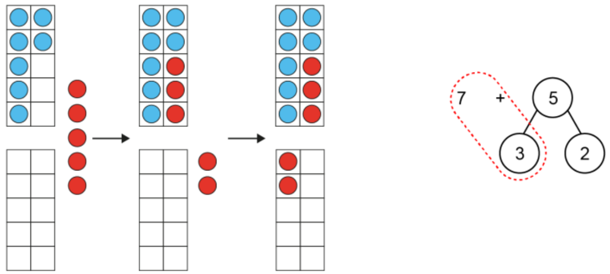


Figure 45: tens frames with counters, and a partitioning diagram, showing $7+5=12$

Pupils can subtract across 10 by using:

- the 'subtracting through 10' strategy (partitioning the subtrahend) – part of the subtrahend is subtracted to reach 10, then the rest of the subtrahend is subtracted from 10 or
- the 'subtracting from 10' strategy (partitioning the minuend) – the subtrahend is subtracted from 10, then the difference between the minuend and 10 is added

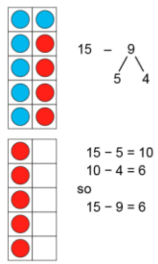


Figure 46: using the 'subtracting through 10' strategy to calculate 15 minus 9

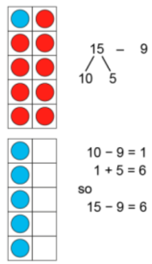


Figure 47: using the 'subtracting from 10' strategy to calculate 15 minus 9

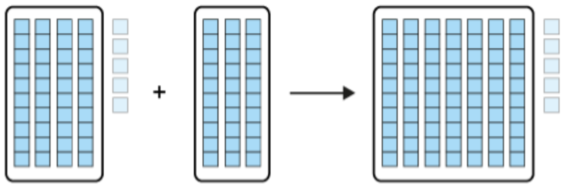
Year 2

2AS–3 Add and subtract within 100 – part 1

Pupils should be able to apply known one-digit additive facts to:

- adding and subtracting 2 multiples of ten (for example, 40 + 30 = 70 and 70 – 30 = 40)
- adding and subtracting ones to/from a two-digit number (for example, 64 + 3 = 67 and 67 – 3 = 64)
- adding and subtracting multiples of ten to/from a two-digit number (for example, 45 + 30 = 75 and 75 – 30 = 45)
- the special case of subtracting ones from a multiple of ten, by using complements of 10 (for example 30 – 3 = 27)

Tens frames, Dienes and partitioning diagrams can be used to support pupils as they learn how to relate these calculations to one-digit calculations.



	4	+	3	=	7
so	40	+	30	=	70
	45	+	30	=	75

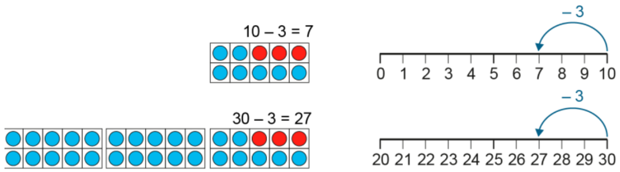


Figure 50: tens frames with counters, and number lines, to support subtracting ones from a multiple of 10

Year 2

2AS–4 Add and subtract within 100 – part 2

Dienes and partitioning diagrams can be used to support pupils as they learn about strategies for carrying out these calculations. To add 2 two-digit numbers, pupils need to combine one-digit addition facts with their understanding of two-digit place value. Pupils should first learn to add 2 multiples of ten and 2 ones before moving on to the addition of 2 two-digit numbers, for example:

- $40+20+5+3=60+8=68$
- $40+5+20+3=60+8=68$
- $45+23=60+8=68$

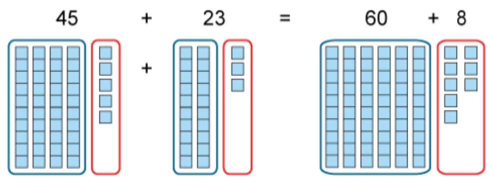
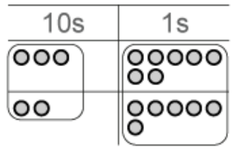


Figure 51: Dienes and an equation to support adding 2 two-digit numbers

When pupils learn to subtract one two-digit number from another, the progression is similar to that for addition. Pupils can first learn to subtract a multiple of ten and some ones from a two-digit number, and then connect this to the subtraction of one two-digit number from another, for example:

26 + 37 = 63 20 6 30 7 20 + 30 = 50 6 + 7 = 13 50 + 13 = 63 Figure 57: adding 26 and 37 by partitioning both addends	26 + 37 = 63 30 7 26 + 30 = 56 56 + 7 = 63 Figure 58: adding 26 and 37 by partitioning one addend
63 – 17 = 46 10 7 63 – 10 = 53 53 – 7 = 46 Figure 59: subtracting 17 from 63 by subtracting the tens first	63 – 17 = 46 10 7 63 – 7 = 56 56 – 10 = 46 Figure 60: subtracting 17 from 63 by subtracting the ones first



$37 + 26 = \boxed{63}$
$30 + 20 = 50$
$7 + 6 = 13$
$50 + 13 = 63$

Pupils do not need to learn formal written methods for addition and subtraction in year 2, but column addition and column subtraction could be used as an alternative way to record two-digit calculations at this stage. For calculations such as 26 + 37, pupils can begin to think about the 2 quantities arranged in columns under place-value headings of tens and ones. They can use counters or draw dots for support.

Year 3

3NF–1 I have secured fluency in addition and subtraction facts that bridge 10, through continued practice.

Identifying core number facts: columnar addition	Identifying core number facts: columnar subtraction
$\begin{array}{r} 4\ 6\ 5 \\ +\ 4\ 2\ 9 \\ \hline 8\ 9\ 4 \\ {\scriptstyle 1} \end{array}$ Figure 72: columnar addition of 465 and 429	$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}4\ 9 \\ -\ 2\ 8\ 6 \\ \hline 4\ 6\ 3 \end{array}$ Figure 73: columnar subtraction of 286 from 749
Within-column calculations: $5+9=14$ $6+2+1=9$ $4+4=8$	Within-column calculations: $9-6=3$ $7-1=6$ $14-8=6$ $6-2=4$

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

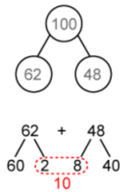
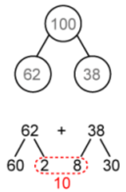
Before pupils begin work on columnar addition and subtraction (3AS–1), it is essential that pupils have automatic recall of addition and subtraction facts within and across 10. These facts are required for calculation within the columns in columnar addition and subtraction. Pupils should practise until they achieve automaticity in the mental application of these strategies.

- **Doubles and near doubles across 10 (3NF-1)**
- **Addition across 10 (3NF-1)**
- **Subtraction across 10 (3NF-1)**

Year 3

3AS–1 Calculate complements to 100

Pupils should compare correct calculations with the corresponding common incorrect calculations for complements to 100. They should be able to discuss the pairs of calculations and understand the source of the error in the incorrect calculations.

Incorrect complement to 100	Correct complement to 100
Figure 77: partitioning diagram and calculation showing incorrect complement to 100: 62 and 48	Figure 78: partitioning diagram and calculation showing correct complement to 100: 62 and 38

A shaded 100 grid can be used to show why there are only 9 full tens in the correct complements to 100. The 10th ten is composed of the ones digits.

Once pupils understand why given complements to 100 are correct or not, they should learn to work in steps to calculate complements themselves:

1. First make 10 ones.
2. Then work out the number of additional tens needed. Pupils must understand that the tens digits should bond to 9, not to 10.
3. Check that the 2 numbers sum to 100.

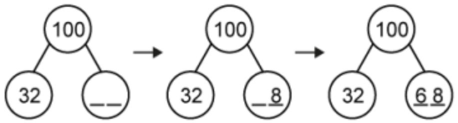


Figure 80: calculating a complement to 100

Year 3

3AS–2 Columnar addition and subtraction

Pupils must learn to add and subtract using the formal written methods of columnar addition and columnar subtraction. Pupils should master columnar addition, including calculations involving regrouping (some columns sum to 10 or more), before learning columnar subtraction. However, guidance here is combined due to the similarities between the two algorithms.

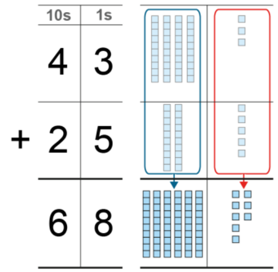


Figure 81: columnar addition with no regrouping: calculation and Dienes representation

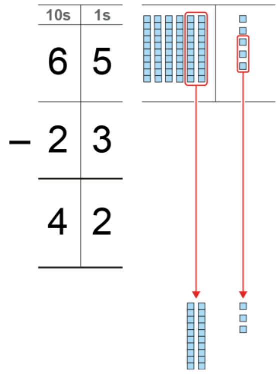


Figure 82: columnar subtraction with no exchange: calculation and Dienes representation

Beginning with calculations that do not involve regrouping (no columns sum to 10 or more) or exchange (no columns have a minuend smaller than the subtrahend), pupils should:

- learn to lay out columnar calculations with like digits correctly aligned
- learn to work from right to left, adding or subtracting the least significant digits first

Teachers should initially use place-value equipment, such as Dienes, to model the algorithms and help pupils make connections to what they already know about addition and subtraction.

Pupils must also learn to carry out columnar addition calculations that involve regrouping, and columnar subtraction calculations that involve exchange.

Year 3

3AS–2 Columnar addition and subtraction (continued)

Dienes (or any other place-value apparatus) should be used, only initially, to support pupils understanding of the structure of the algorithms, and should not be used as a tool for finding the answer.

Pupils must be able to add 2 or more numbers using columnar addition, including calculations whose addends have different numbers of digits.

$$\begin{array}{r} 2\,7\,4 \\ +\ 3\,5\,4 \\ \hline 6\,2\,8 \\ {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 6\,2 \\ +\ 4\,8\,1 \\ \hline 5\,4\,3 \\ {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 1\,8\,6 \\ 5\,7 \\ +\ 4\,3\,4 \\ \hline 6\,7\,7 \\ {\scriptstyle 1\ 1} \end{array}$$

Figure 101: columnar addition for calculations involving three-digit numbers

Pupils must be able to subtract 1 three-digit number from another using columnar subtraction. They should be able to apply the columnar method to calculations where the subtrahend has fewer digits than the minuend, and they must be able to exchange through 0.

$$\begin{array}{r} \overset{5}{\cancel{6}}\,{}^{1}2\,8 \\ -\ 2\,7\,4 \\ \hline 3\,5\,4 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{14}{\cancel{5}}\,{}^{1}6 \\ -\ 7\,8 \\ \hline 4\,7\,8 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}\,\overset{{}^{1}9}{\cancel{0}}\,{}^{1}2 \\ -\ 1\,5\,4 \\ \hline 1\,4\,8 \end{array}$$

Figure 102: columnar subtraction for calculations involving three-digit numbers

Throughout, pupils should continue to recognise the inverse relationship between addition and subtraction. Pupils may represent calculations using partitioning diagrams or bar models, and should learn to check their answers using the inverse operation.

$$\begin{array}{r} \overset{5}{\cancel{6}}\,{}^{1}2\,8 \\ -\ 2\,7\,4 \\ \hline 3\,5\,4 \end{array}$$

628	
274	354

$$\begin{array}{r} 2\,7\,4 \\ +\ 3\,5\,4 \\ \hline 6\,2\,8 \\ {\scriptstyle 1} \end{array}$$

Figure 84: using addition to check a subtraction calculation

Year 3

3AS–3 Manipulate the additive relationship

Pupils must understand that the simplest addition and subtraction equations describe the relationship between 3 numbers, where one is a sum of the other two. They should understand that both addition and subtraction equations can be used to describe the same additive relationship. They should practise writing the full set of 8 equations that are represented by a given partitioning diagram or bar model.

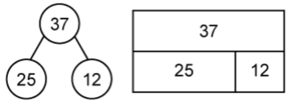


Figure 85: partitioning diagrams showing the additive relationship between 25, 12 and 37

$25+12=37$ $\quad 37-12=25$
$12+25=37$ $\quad 37-25=12$

$37=25+12$ $\quad 25=37-12$
$37=12+25$ $\quad 12=37-25$

Pupils should learn and use the correct names for the terms in addition and subtraction equations.

$$\text{addend}+\text{addend}=\text{sum}$$
$$\text{minuend}-\text{subtrahend}=\text{difference}$$

Pupils understanding should go beyond the fact that addition and subtraction are inverse operations. They need understand how the terms in addition and subtraction equations are related to each other, and to the parts and whole within an additive relationship, and use this understanding to manipulate equations.

sum	
addend	addend

minuend	
subtrahend	difference

minuend	
difference	subtrahend

Figure 86: connecting the terms in addition and subtraction equations to the part-part-whole structure

With experience of the commutative property of addition, pupils can now learn that, because of the relationship between addition and subtraction, the commutative property has a related property for subtraction. Pupils need to be able to solve:

- missing-addend problems (addend + ? = sum)
- missing-subtrahend problems (minuend - ? = difference)
- missing-minuend problems (? - subtrahend = difference)

Year 4

Addition and subtraction: extending 3AS–3

Pupils should also extend columnar addition and subtraction methods to four-digit numbers. Pupils must be able to add 2 or more numbers using columnar addition, including calculations whose addends have different numbers of digits.

$$\begin{array}{r} 6,5\,8\,4 \\ +\ 2,7\,3\,9 \\ \hline 9,3\,2\,3 \\ {\scriptstyle 1\ 1\ 1} \end{array} \qquad \begin{array}{r} 3,3\,6\,2 \\ +\ 6\,4\,9 \\ \hline 4,0\,1\,1 \\ {\scriptstyle 1\ 1\ 1} \end{array} \qquad \begin{array}{r} 1,6\,4\,9 \\ 3,1\,0\,4 \\ +\ 5\,1\,6 \\ \hline 5,2\,6\,9 \\ {\scriptstyle 1\ \ \ 1} \end{array}$$

Figure 150: columnar addition for calculations involving four-digit numbers

Pupils must be able to subtract one four-digit number from another using columnar subtraction. They should be able to apply the columnar method to calculations where the subtrahend has fewer digits than the minuend, and must be able to exchange through 0.

$$\begin{array}{r} \overset{5}{\cancel{6}},\overset{14}{\cancel{5}}\,\overset{12}{\cancel{3}}\,{}^{1}8 \\ -\ 2,7\,8\,9 \\ \hline 3,7\,4\,9 \end{array} \qquad \begin{array}{r} 2,7\,9\,6 \\ -\ 4\,8\,5 \\ \hline 2,3\,1\,1 \end{array} \qquad \begin{array}{r} 8,\overset{3}{\cancel{4}}\,\overset{{}^{1}9}{\cancel{0}}\,{}^{1}3 \\ -\ 2,1\,7\,6 \\ \hline 6,2\,2\,7 \end{array}$$

Figure 151: columnar subtraction for calculations involving four-digit numbers

Pupils should make sensible decisions about how and when to use columnar subtraction. For example, when the minuend is a multiple of 1,000, they may transform to an equivalent calculation before using column subtraction, avoiding the need to exchange through zeroes.

$$\begin{array}{r} 7,0\,0\,0 \\ -\ 2,6\,4\,8 \\ \hline \\ \hline \end{array} \quad \begin{array}{c} \xrightarrow{-1} \\ \xrightarrow[-1]{} \end{array} \quad \begin{array}{r} 6,9\,9\,9 \\ -\ 2,6\,4\,7 \\ \hline \\ \hline \end{array}$$

Figure 152: transforming a columnar subtraction calculation to an equivalent calculation

Year 5

Addition and subtraction: extending 3AS–3

Pupils should also extend columnar addition and subtraction methods to numbers with up to 2 decimal places. Pupils must be able to add 2 or more numbers using columnar addition, including calculations whose addends have different numbers of digits.

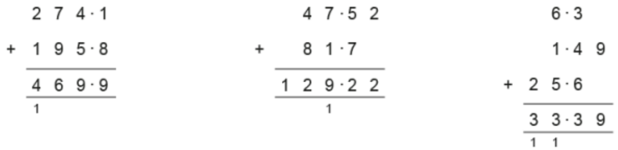


Figure 202: columnar addition for calculations involving numbers with up to 2 decimal places

Pupils must be able to subtract one number from another using columnar subtraction, including numbers with up to 2 decimal places. They should be able to apply the columnar method to calculations presented as, for example, 21.8 – 9.29 or 58 14.69 – , where the subtrahend has more decimal places than the minuend. Pupils must also be able to exchange through 0.

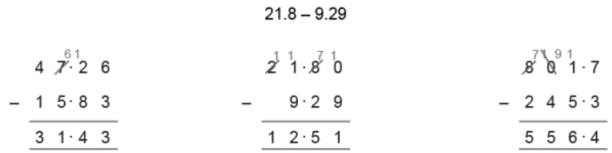


Figure 203: columnar subtraction for calculations involving numbers with up to 2 decimal places

Year 6

Addition and subtraction: formal written methods

Pupils should continue to practise adding whole numbers with up to 4 digits, and numbers with up to 2 decimal places, using columnar addition. This should include calculations with more than 2 addends, and calculations with addends that have different numbers of digits.

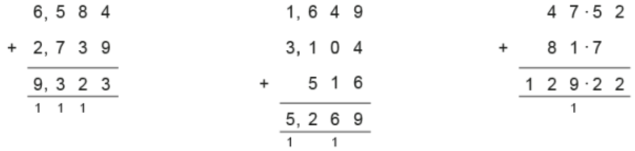


Figure 237: range of columnar addition calculations

Pupils should continue to practise using columnar subtraction for numbers with up to 4 digits, and numbers with up to 2 decimal places. This should include calculations where the minuend and subtrahend have a different numbers of digits or decimal places, and those involving exchange through 0.

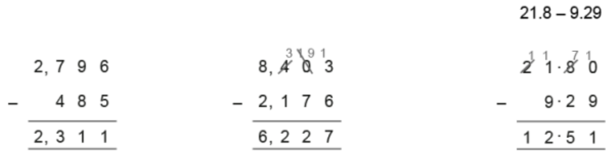


Figure 238: range of columnar subtraction calculations

Pupils should make sensible decisions about how and when to use columnar methods. For example, when subtracting a decimal fraction from a whole number, pupils may be able to use their knowledge of complements, avoiding the need to exchange through zeroes. For example, to calculate 8 – 4.85 pupils should be able to work out that the decimal complement to 5 from 4.85 is 0.15, and that the total difference is therefore 3.15.

Multiplication and Division

Year 1

1NF–2 Count forwards and backwards in multiples of 2, 5 and 10

Pupils must be able to count in multiples of 2, 5 and 10 by the end of year 1 so that they are ready to progress to multiplication involving groups of 2, 5 and 10 in year 2.

Forwards and backwards counting practice should include:

- reciting just the number names (for example, “ten, twenty, thirty…”), without the support of visual representations
- counting with the support of visual representations and gestural patterns, for example pupils can point to numerals on a number line or 100 square, or tap out the numbers on a Gattegno chart
- starting the forwards counting sequence with numbers other than 2, 5 or 10

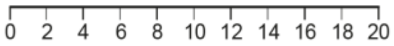


Figure 12: number line to support counting in multiples of 2

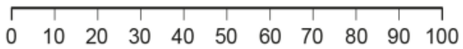


Figure 13: number line to support counting in multiples of 10

When pupils are confident with the counting sequences, they should learn to enumerate objects arranged in groups of 2, 5 or 10 by skip counting, so they begin to connect counting in multiples with finding the total quantity for repeated groups. Pupils should first identify how many objects are in each repeated group, and then skip count in this number.

How many pencils are there?

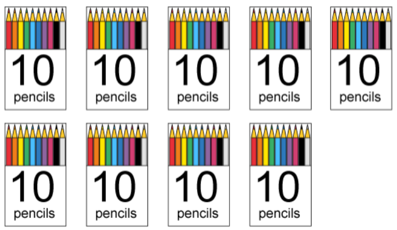


Figure 15: counting in tens – 9 packs of 10 pencils

Once pupils have learnt to recognise 2p, 5p and 10p coins, they should be expected to apply their knowledge of counting in multiples of 2, 5 and 10 to find the value of a set of like coins, and to find how many of a particular denomination is required to pay for a given item.

Pupils should also learn to recite the odd number sequence, both forwards and backwards. This can initially be supported by a number line with odd numbers highlighted.

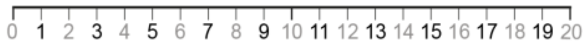


Figure 16: number line to support counting through the odd number sequence

Year 2

2MD–1 Multiplication as repeated addition

Pupils must first be able to recognise equal groups. To better understand and identify equal groups, pupils should initially explore both equal and unequal groups. Pupils should then learn to describe equal groups with words.

Figure 52: recognising equal groups – 3 groups of 5 eggs

Based on their existing additive knowledge, pupils should be able to represent equal group contexts with repeated addition expressions, for example 5+5+5. They should then learn to write multiplication expressions to represent the same contexts, for example 3×5 . Pupils must also be able to understand equivalence between a repeated addition expression and a multiplication expression: $5 + 5 + 5 = 3 \times 5$.

Pupils should be able to relate multiplication to situations where the total number of items cannot be seen, for example by representing 3 5× with three 5-value counters.

Figure 53: three 5-value counters

Year 2

2MD–2 Grouping problems: missing factors and division

Pupils need to be able to represent problems where the total quantity and group size is known, using multiplication equations with missing factors. For example, “There are 15 biscuits. If I put them into bags of 5, how many bags will I need?’’ can be represented by the following equation:

$$\square \times 5 = 15$$

Pupils can use skip counting or their emerging 2, 5 and 10 multiplication table fluency to calculate the missing factor.

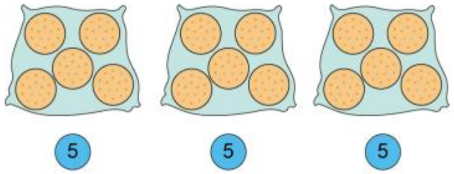


Figure 54: 3 bags of 5 biscuits alongside three 5-value counters

Pupils also need to be able to solve division calculations that are not set in contexts. They should recognise that they need to skip count in the divisor (2, 5 or 10), or use the associated multiplication fact, to find the quotient. For example, to calculate $60 \div 10$, they can skip count in tens (counting the required number of tens) or apply the fact that $6 \times 10 = 60$.

Year 3

3NF–2 Recall of multiplication tables

The national curriculum requires pupils to recall multiplication table facts up to 12×12, and this is assessed in the year 4 multiplication tables check. In year 3, the focus should be on learning facts in the 10, 5, 2, 4 and 8 multiplication tables.

While pupils are learning the individual multiplication tables, they should also learn that:

- the factors can be written in either order and the product remains the same (for example, we can write $3 \times 4 = 12$ or $4 \times 3 = 12$ to represent the third fact in the 4 multiplication table)
- adjacent multiples in, for example, the 8 multiplication table, have a difference of 8

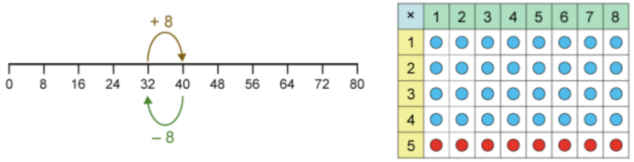


Figure 74: number line and array showing that adjacent multiples of 8 (32 and 40) have a difference of 8

As pupils develop automatic recall of multiplication facts, they should learn how to use these to derive division facts. They must be able to use these ‘known division facts’ to solve division calculations, instead of using the skip counting method they learnt in year 2.

It is useful for pupils to learn the 10 and 5 multiplication tables one after the other, and then the 2, 4 and 8 multiplication tables one after the other. The connections and patterns will help pupils to develop fluency and understanding.

Year 3

3MD–1 Multiplication and division structures

Pupils should also learn that the commutative property allows them to use their known facts to solve problems about 5, 10, 2, 4 or 8 equal groups (for example, 2 groups of 7). An array can be used to illustrate how the commutative property relates to different grouping interpretations – the example below shows that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity (14).

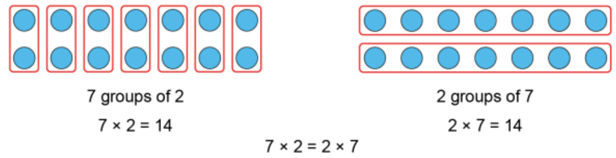


Figure 87: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity

This means that pupils can use their knowledge that $7 \times 2 = 14$ to solve a problem about 2 groups of 7, even though they have not yet learned the 7 multiplication table.

Year 3	Year 4	Year 4
3MD–1 Multiplication and division structures	**4NF–1 Recall of multiplication tables**	**4NF–2 Division problems with remainders**

Year 3

3MD–1 Multiplication and division structures

Pupils should already be solving division calculations using known division facts corresponding to the 5, 10, 2, 4 and 8 multiplication tables (3NF–2). They must also be able to use these known facts to solve both quotitive (grouping) and partitive (sharing) contextual division problems.

Quotitive division

I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?

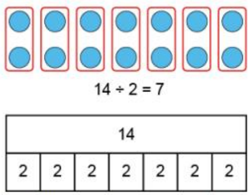


Figure 88: using an array and bar model to show that 14 divided into groups of 2 is equal to 7

Language focus

"7 times 2 is 14, so 14 divided by 2 is 7."

"14 divided into groups of 2 is equal to 7."

I need 7 packs of ping-pong balls.

Partitive division

£14 is shared between 2 children. How much money does each child get?

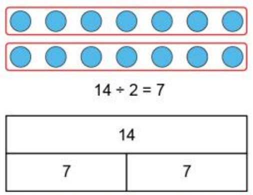


Figure 89: using an array and bar model to show that 14 shared between 2 is equal to 7

Language focus

"7 times 2 is 14, so 14 divided by 2 is 7."

"£14 shared between 2 is equal to £7 each."

Each child gets £7.

At this stage, pupils only need to be able to apply division facts corresponding to division by 5, 10, 2, 4 and 8 to solve division problems with the two different contexts. In year 4, pupils will learn, for example, that if they know that $2 \times 7 = 14$, then they know both $14 \div 2 = 7$ and $14 \div 7 = 2$.

Year 4

4NF–1 Recall of multiplication tables

The 36 multiplication facts that are required for formal written multiplication are as follows.

2 × 2 = 4							
2 × 3 = 6	3 × 3 = 9						
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16					
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25				
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30	6 × 6 = 36			
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35	6 × 7 = 42	7 × 7 = 49		
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81

During application of formal written multiplication, pupils may also need to multiply a one-digit number by 1. Multiplication of the numbers 1 to 9 by 1 are not listed here because these calculations do not need to be recalled in the same way.

It is useful for pupils to learn the multiplication tables in the following order/groups:

- 10 then 5 multiplication tables
- 2, 4 and 8 multiplication tables one after the other
- 3, 6, and 9 multiplication tables one after the other
- 7 multiplication table
- 11 and 12 multiplication tables

2 × 8 8 × 2	16

During the Mastering Number Programme, pupils should be given the opportunity to make their own set of practice cards. Pupils can add to this set of cards during the programme, as new multiplication facts are learned. On one side of each card, pupils should write two expressions using the same factors; on the reverse, they should write the product (see example on the left). Please note that the smallest factor is always said first, and it should be written first in the first expression on the card. Pupils can use these cards to practise saying and recalling the facts shown, either on their own or with a partner.

Year 4

4NF–2 Division problems with remainders

Pupils should recognise that a remainder arises when there is something 'left over' in a division calculation. Pupils should recognise and understand why remainders only occur when the dividend is not a multiple of the divisor. This can be achieved by discussing the patterns seen when the dividend is incrementally increased by 1 while the divisor is kept the same.

Total number of apples (dividend)	Number of apples in each tray (divisor)	Number of trays (quotient)	Number of apples left over (remainder)	Equation
12	**4**	3	**0**	12 ÷ 4 = 3
13	**4**	3	**1**	13 ÷ 4 = 3 r 1
14	**4**	3	**2**	14 ÷ 4 = 3 r 2
15	**4**	3	**3**	15 ÷ 4 = 3 r 3
16	**4**	4	**0**	16 ÷ 4 = 4
17	**4**	4	**1**	17 ÷ 4 = 4 r 1
18	**4**	4	**2**	18 ÷ 4 = 4 r 2
19	**4**	4	**3**	19 ÷ 4 = 4 r 3
20	**4**	5	**0**	20 ÷ 4 = 5

Once pupils can correctly perform division calculations that involve remainders, they need to recognise that, when solving contextual division problems, the answer to the division calculation must be interpreted carefully to determine how to make sense of the remainder. The answer to the calculation is not always the answer to the contextual problem.

Consider the following context: Four scouts can fit in each tent. How many tents will be needed for thirty scouts?

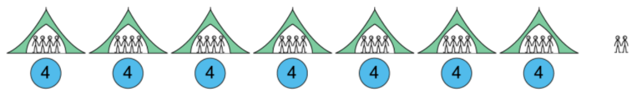


Figure 114: pictorial representation and counters: with 30 scouts and 4 per tent, 7 tents are insufficient

$30 \div 4 = 7 \text{ r } 2$

Year 4

4MD–2 Manipulating the multiplicative relationship

Pupils need to be able to apply the commutativite property of multiplication in 2 different ways. The first can be summarised as ‘1 interpretation, 2 equations’. Here pupils must understand that 2 different equations can correspond to one context, for example, 2 groups of 3 is equal to 6 can be represented by $2\times 3 = 6$ and by $3\times 2 = 6$. Spoken language can support this understanding.

The second way that pupils must understand commutativity, can be summarised as ‘one equation, two interpretations’. Here, pupils must understand that a single equation, such as $2\times 7 = 14$, can be interpreted in two ways.

An array is an effective way to illustrate this.

7 groups of 2

$7 \times 2 = 14$

2 groups of 7

$2 \times 7 = 14$

$7 \times 2 = 2 \times 7$

Figure 120: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity

Pupils must be able to describe what each number in the equation represents for the 2 different interpretations, in context.

$$7\times 2=14$$

$$2\times 7=14$$

Pupils need to be able to fluently move between the different equations in a set, and understand how one known fact (such as $7 \times 2 = 14$) allows them to solve 4 different calculations each with two possible interpretations.

Interpretation 1

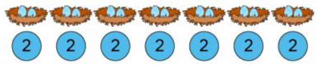


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

Language focus

“The 2 represents number of eggs in each nest/group”.

“The 7 represents the number of nests/groups.”

“The 14 represents the total number of eggs/product.”

Interpretation 2

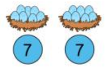


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

Language focus

“The 2 represents the number of nests/groups.”

“The 7 represents the number of eggs in each nest/group.”

“The 14 represents the total number of eggs/product.”

Pupils must understand that, because division is the inverse of multiplication, any multiplication equation can be rearranged to give division equations. The value of the product in the multiplication equation becomes the value of the dividend in the corresponding division equations.

$$2\times 7=14 \qquad 7\times 2=14 \qquad 14\div 2=7 \qquad 14\div 7=2$$

As with multiplication, any division equation can be interpreted in two different ways, and these correspond to quotitive and partitive division.

$$14\div 7=2$$

Partitive division

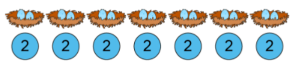


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

Language focus

“14 shared between 7 is equal to 2.”

“The 14 represents the total number of eggs.”

“The 7 represents the number of nests/shares.”

“The 2 represents the number of eggs in each nest/share.”

Quotitive division

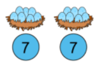


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

Language focus

“14 divided into groups of 7 is equal to 2.

“The 14 represents the total number of eggs.”

“The 7 represents the number of eggs in each nest/group.”

“The 2 represents the number of nests/groups.”

Year 4

4MD–3 The distributive property of multiplication

The first place that pupils will have encountered the distributive law is within the multiplication tables themselves. Pupils have seen, for example, that adjacent multiples in the 6 times table have a difference of 6. Number lines and arrays can be used to illustrate this.

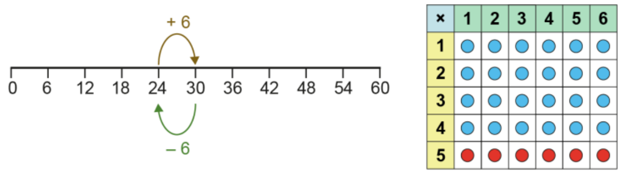


Figure 124: number line and array showing that adjacent multiples of 6 (24 and 30) have a difference of 6

Pupils should be able to represent such relationships using mixed operation equations, for example:

$$5\times 6=4\times 6+6 \quad \text{or} \quad 5\times 6=4\times 6+1\times 6$$

$$4\times 6=5\times 6-6 \quad \text{or} \quad 4\times 6=5\times 6-1\times 6$$

Pupils should learn that multiplication takes precedence over addition. They should then extend this understanding beyond the multiplication tables, for example, if they are given the equation $20\times 6=120$, they should be able to determine that $21\times 6=126$, or vice versa. Pupils also need to be able to apply the distributive property to non-adjacent multiples. The array chart used to show the connection between 4×6 and 5×6 can be adapted to show the connection between 5×6, 3×6 and 2×6.

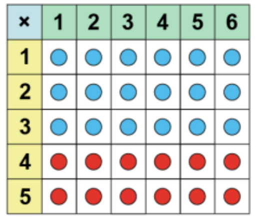


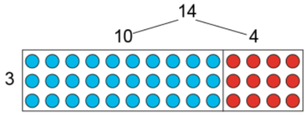
Figure 125: Array showing the relationship between 5×6, 3×6 and 2×6

Year 4	Year 5	Year 5
4MD–3 The distributive property of multiplication	**5NF–1 Secure fluency in multiplication and division facts**	**5MD–2 Find factors and multiples**

Year 4

4MD–3 The distributive property of multiplication

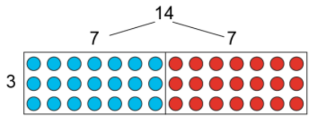
Pupils should then use the distributive property and known multiplication table facts to multiply 2-digit numbers (above 12) by one-digit numbers. 14×3 , for example, can be calculated by relating it to 10×3 and 4×3 :



$$\begin{aligned} 14\times 3 &= 10\times 3+4\times 3 \\ &= 30+12 \\ &= 42 \end{aligned}$$

Figure 126: array showing the relationship between 14×3, 10×3 and 4×3

Pupils should recognise that factors can be partitioned in ways other than into ‘10 and a bit’. For example 14×3 could also be related to double 7×3.



$$\begin{aligned} 14\times 3 &= 7\times 3+7\times 3 \\ &= 21+21 \\ &= 42 \end{aligned}$$

Figure 127: array showing that 14×3 is double 7×3

Pupils should be expected to extend their understanding of the distributive law to support division. For example, they should be able to connect a calculation such as $65 \div 5$ to known multiplication and division facts: “we have 3 more fives than 10 fives” or “we have 1 more 5 than 12 fives”.

Year 5

5NF–1 Secure fluency in multiplication and division facts

Before pupils begin work on formal multiplication and division (5MD–3 and 5MD–4), it is essential that pupils have automatic recall of multiplication and division facts within the multiplication tables. These facts are required for calculation within the ‘columns’ during application of formal written methods. All mental multiplicative calculation also depends on these facts.

Identifying core number facts: short multiplication	Identifying core number facts: short division
$\begin{array}{r} 3\;4\;2 \\ \times\quad\;\; 7 \\ \hline 2,3\;9\;4 \\ {\scriptstyle 2\;\;1\;\;} \end{array}$ **Figure 170: short multiplication of 342 by 7**	$\begin{array}{r} 6\;1\;9 \\ 8\,\overline{)\,4,{}^{4}9\;{}^{1}5\;{}^{7}2} \end{array}$ **Figure 171: short division of 4,952 by 8**
Within-column calculations: $7 \times 2 = 14$ $7 \times 4 + 1 = 28 + 1$ $7 \times 3 + 2 = 21 + 2$	Within-column calculations: $4 \div 8 = 0$ r 4 $49 \div 8 = 6$ r 1 $15 \div 8 = 1$ r 7 $72 \div 8 = 9$

Pupils should already have automatic recall of multiplication table facts and corresponding division facts, from year 3 (5, 10, 2, 4 and 8 multiplication tables, 3NF–2) and year 4 (all multiplication tables up to and including 12, 4NF–1). Pupils’ fluency in multiplication facts is assessed in the summer term of year 4 in the statutory multiplication tables check, and this will identify some pupils who need additional practice. Pupils must also be able to fluently derive related division facts, including division facts with remainders before they begin to learn formal written methods for multiplication and division (5MD–3 and 5MD–4).

Year 5

5MD–2 Find factors and multiples

In year 5, pupils should learn the definitions of the terms ‘multiple’ and ‘factor’, and understand the inverse relationship between them.

Pupils must be able to identify factors and multiples within the multiplication tables, and should learn to work systematically to identify all of the factors of a given number. They should be able to express products in the multiplication tables as products of 3 factors, where relevant, for example, $48= 2 \times 3 \times 8$.

Pupils should learn to express multiples of 10 or 100 as products of 3 factors, for example:
$7 \times 3 =21$ so $7\times 3 \times 10 =210$

Pupils should learn that these factors can be written in any order (commutative property of multiplication) and that any pair of the factors can be multiplied together first (associative property of multiplication).

Applying commutativity		Applying associativity (example)
$3\times 7\times 10=210$	$3\times 10\times 7=210$	$3\times 7\times 10=210$
$7\times 3\times 10=210$	$7\times 10\times 3=210$	$(3\times 7)\times 10=21\times 10=210$
$10\times 3\times 7=210$	$10\times 7\times 3=210$	$3\times (7\times 10)=3\times 70=210$

Pupils should learn to identify factors and multiples for situations other than those described above by using short division or divisibility rules. For example, to determine whether 392 is a multiple of 8 (or whether 8 is a factor of 392) pupils can use the divisibility rule for 8 or use short division to determine whether $392\div 8$ results in a quotient without a remainder.

Pupils must learn how to find common factors and common multiples of small numbers in preparation for simplifying fractions and finding common denominators. They must also learn to recognise and use squared numbers and use the correct notation (for example, $3^2 = 9$), and learn to establish whether a given number (up to 100) is prime.

Year 5

5MD–3 Multiply using a formal written method

Pupils must be able to multiply whole numbers with up to 4 digits by one-digit numbers using short multiplication.

Pupils should learn to multiply multi-digit numbers by one-digit numbers using the formal written method of short multiplication. They should begin with examples that do not involve regrouping, such as the two-digit one-digit × calculation shown below, and learn that, like columnar addition and subtraction, the algorithm begins with the least significant digit (on the right). When pupils first learn about short multiplication, place-value equipment (such as place-value counters) should be used to model the algorithm and help pupils relate it to what they already know about multiplication.

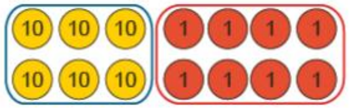


Figure 179: place-value counters showing 34 × 2

Informal written method	Expanded multiplication algorithm	Short multiplication
$34 \times 2 = 30 \times 2 + 4 \times 2$ $= 60 + 8$ $= 68$	10s 1s 3 4 × 2 8 2 × 4 ones = 8 ones 6 0 2 × 3 tens = 6 tens 6 8	10s 1s 3 4 × 2 6 8

Pupils may also use place-value headings while they learn to use the formal written method, as illustrated above. However, by the end of year 5, they must be able to use the short multiplication algorithm without using unitising language and place-value headings.

Once pupils have mastered the basic principles of short multiplication without regrouping, they must learn to use the algorithm where regrouping is required, for multiplication of numbers with up to 4 digits by one-digit numbers.

```
    3 6 7
  ×     4
  1,4 6 8
     2 2
```

Figure 180: multiplying 367 by 4 using short multiplication

4×7 ones	= 28 ones
	= 2 tens + 8 ones
4×6 tens	= 24 tens
	= 2 hundreds + 4 tens
plus 2 more tens	= 2 hundreds + 6 tens
4×3 hundreds	= 12 hundreds
	= 1 thousand + 2 hundreds
plus 2 more hundreds	= 1 thousand + 4 hundreds

Once pupils have mastered the basic principles of short multiplication without regrouping, they must learn to use the algorithm where regrouping is required, for multiplication of numbers with up to 4 digits by one-digit numbers.

Pupils must be able to use short multiplication to solve contextual multiplication problems with:

- the grouping structure
- the scaling structure

Pupils should also be able to use short multiplication to solve missing-dividend problems (for example, ? ÷2,854 =3)

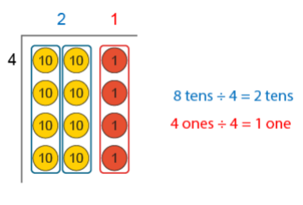
```
    2 4          3 4 2        2,3 7 1
  ×   6        ×     7      ×       4
  1 4 4        2,3 9 4      9,4 8 4
    2            2 1          1 2
```

Figure 204: short multiplication for multiplication of 2-, 3- and 4-digit numbers by one-digit numbers

Year 5

5MD–4 Divide using a formal written method

Pupils should learn to divide multi-digit numbers by one-digit numbers using the formal written method of short division. They should begin with examples that do not involve exchange, such as the two-digit one-digit ÷ calculation shown below. Pupils should learn that division is the only operation for which the formal algorithm begins with the most significant digit (on the left). When pupils first learn about short division, place-value equipment (such as place-value counters) should be used to model the algorithm and help pupils relate it to what they already know about division.

Short division with place-value counters	Short division
 Figure 181: dividing 84 by 4 using short division with place-value counters	10s 1s 2 1 4 ⟌ 8 4 Figure 182: dividing 84 by 4 using short division with place-value headings

Pupils may also use place-value headings while they learn to use the formal written method, as illustrated above. However, by the end of year 5, pupils must be able to use the short division algorithm without using unitising language and place-value headings.

Once pupils have mastered the basic principles of short division without exchange, they must learn to use the algorithm where exchange is required, for division of numbers with up to 4 digits by one-digit numbers.

```
   1 5 3
4 ⟌ 6 ²1 ¹2
```

Figure 183: dividing 612 by 4 using short division

6 hundreds ÷ 4	= 1 hundred remainder 2 hundreds
2 hundreds	= 20 tens
plus 1 more ten	= 21 tens
21 tens ÷ 4	= 5 tens remainder 1 ten
1 ten	= 10 ones
plus 2 more ones	= 12 ones
12 ones ÷ 4	= 3 ones

Year 5

5MD–4 Divide using a formal written method

Pupils must be able to divide numbers with up to 4 digits by one-digit numbers using short division, including calculations that involve remainders. Pupils do not need to be able to express remainders arising from short division, using proper fractions or decimal fractions.

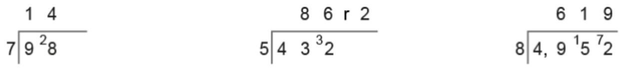


Figure 205: short division for division of 2-, 3- and 4-digit numbers by one-digit numbers

Pupils should be fluent in interpreting contextual problems to decide when division is the appropriate operation to use, including as part of multi-step problems. Pupils should use short division when appropriate to solve these calculations. For contextual problems, pupils must be able to interpret remainders appropriately as they learnt to do in year 4.

Year 6

Multiplication: extending 5MD–3

In year 5, pupils learnt to multiply any whole number with up to 4 digits by any 1-digit number using short multiplication (5MD–3). They should continue to practise this in year 6. Pupils should also learn to use short multiplication to multiply decimal numbers by 1-digit numbers, and use this to solve contextual measures problems, including those involving money.

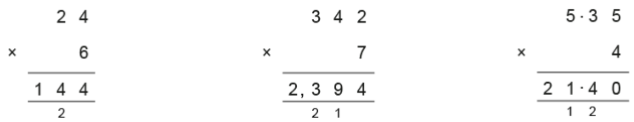


Figure 239: range of short multiplication calculations

Pupils should be able to multiply a whole number with up to 4 digits by a 2-digit whole number by applying the distributive property of multiplication (4MD–3). This results in multiplication by a multiple of 10 and multiplication by a one-digit number.

$$\begin{aligned} 124\times 26 &= 124\times 20+124\times 6 \\ &= 124\times 2\times 10+124\times 6 \\ &= 2{,}480+744 \\ &= 3{,}224 \end{aligned}$$

Pupils should be able to represent this using the formal written method of long multiplication, and explain the connection to the partial products resulting from application of the distributive law.

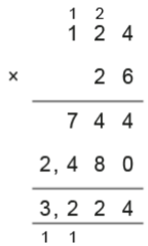


Figure 240: long multiplication calculation

Pupils should learn to check their short and long multiplication calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.

Year 6

Division: extending 5MD–4

In year 5, pupils learnt to divide any whole number with up to 4 digits by a 1-digit number using short division, including with remainders (5MD–4). They should continue to practise this in year 6. Pupils should also learn to use short division to express remainders as a decimal fraction.

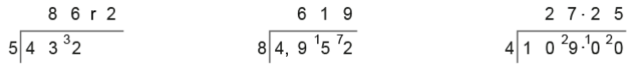


Figure 241: range of short division calculations

Pupils should also be able to divide any whole number with up to 4 digits by a 2-digit number, recording using either short or long division. Pupils are likely to need to write out multiples of the divisor to carry out these calculations and can do this efficiently using a ratio table – they can write out all multiples up to 10× (working in the most efficient order) or write out multiples as needed.

	×17
1	17
2	34
3	51
4	68
5	85
6	
7	
8	136

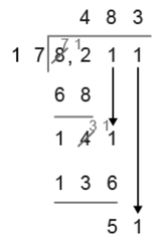


Figure 242: long division calculation
($8{,}211\div 17$)

Pupils should be fluent in interpreting contextual problems to decide when division is the appropriate operation to use, including as part of multi-step problems. Pupils should use short or long division as appropriate to solve these calculations.

Pupils should learn to check their short and long division calculations with a calculator so that they know how to use one. This will help pupils when they progress to key stage 3.